CONGESTION CHARGE AND RETURN SCHEMES ON MODAL CHOICE BETWEEN ROAD AND RAILROAD

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Abstract: Recently, "Congestion charging" policy is considered as the quickest and most effective policy to solve traffic congestion problem, and some metropolises have introduced it into their downtown roads. Most of these metropolises have alternate transport systems, such as subways or commuter rail, then the congestion charging influences modal split. It is also important to decide how to return the revenue from the charging to the commuters, because return policy affects on the agreement of the commuters. In this paper, we propose and solve optimal control models simultaneously determining the distribution of home departure time and the modal choice between road and railroad with/ without the congestion charging. We theoretically make clear the optimal congestion charge each time of day for each transport mode. Furthermore, we analyze influence of return scheme on the modal choice. From results of numerical simulations using above models, charging and return to railroad commuters is proved to be effective scheme to improve commuters' utility and to reduce the number of automobile commuters.

Key Words: Charging and return schemes, Modal choice, Commuting time distribution

1. INTRODUCTION

In most Asian metropolises, because road and railroad infrastructures are far from the satisfied level, they are suffering from heavy traffic congestions and environmental problem due to exhaust gas on motor commuting. We should be required to settle these traffic congestion problems quickly in order to reduce economic and environmental loss. However, considering the limit of budget and a long time period for construction, we cannot expect a large amount of traffic infrastructures expansion, any more. Recently, therefore, the "Transportation demand management (TDM)" policies are expected in order to level the peaks of commuting demand temporally. "Staggered work hours" and "Flexible work hours" are considered as more feasible policies from the aspect of introduction and operation costs. These policies have already practiced in social experiments, and we have confirmed that they can reduce traffic congestion, and local governments hope to promote these policies. However, most firms do not introduce these policies, because they suspect decrease of efficiency of business activities. For the reasons mentioned above, "Congestion charging" policy becomes considered as the most promising policy, because we can directly control the commuting demand and compensate the possible loss of business activities by TDM policies above. Some metropolises (e.g. Singapore) have already introduced this policy into their downtown roads. In these metropolises, charge is constant in particular period of time in a day, but variable day by day. We should charge the optimal and different amount on each time of day in a day if we carry out this policy more effective. Most metropolises expecting the congestion charging have alternate transport systems (e.g. commuter train/ subway). Therefore, the congestion charging may influence on modal split between road and railroad. Furthermore, because revenue is occurred by charging, in contrast with other TDM policies, it is important to decide how to return the revenue to the commuters (e.g. the government can use the revenue for improving railroad service level). Theoretically, it is said that we should use the revenue improving the service level of the traffic infrastructure in the long run. However, considering the difficulties for the traffic infrastructure investments, it may be realistic to return the revenue directly to commuters. The electronic toll collecting system enables us to carry out these charging and return schemes easily. If we return the revenue to other mode commuters, we can shift the modal choice of commuters. Then, we should make clear what effects are given by the congestion charging and return schemes on modal choice, in order to operate the congestion charging, successfully.

In this paper, we propose and solve theoretical optimal control models to simultaneously determine the distribution of home departure time and the modal choice between road and railroad with/ without the congestion charging. We theoretically make clear the optimal congestion charge each time of day for each transport mode. Furthermore, we analyze the influence of return scheme on the modal choice between road and railroad in the context of fixed infrastructure levels.

2. RELATED STUDIES UP TO NOW

One of the easiest ways to describe the relationship between the demand and trip time along congested road is queuing theory. A pioneering study utilized queuing theory to describe waiting time at one bottleneck (Vickrey, 1969). When commuters choose their home departure time, they meet with trade-off between travel time and schedule cost, which is related to the difference between desired time and actual departure time. Arnott, R. *et al.* (1990) expanded Vickrey's model, and compared the social optimum and no-toll equilibrium. Furthermore, that study was extended to the situation of two different types of commuters in desired work start time (Arnott, R. *et al.*, 1998), and stochastic capacity and demand in the bottleneck (Arnott, R. *et al.*, 1999).

On the other hand, railroad commuting demand distribution along time has been studied from late 1980's. Ieda *et al.* (1988) developed a logit model to describe the commuters' choice over the trade-off among time, congestion and transfer. Based on that model, they estimated economic evaluation of several service schedule alternatives, but cost structure of railroad company was missed in this study. Kobayashi *et al.* (1997) began to develop a partial equilibrium model to describe the interaction between railroad company and commuters. Because railroad service market is differentiated by departure time, commuters can choose sub-market by deciding his/ her departure time. In the field of urban economics, optimal control model skillfully analyzed spatially differentiated market (Fujita, 1989), that give a good guidance for their modeling.

Because all of above studies only treated single transport system, we cannot apply these models directly to the problem with modal shift. Tabuchi (1993) considered road system with alternate railroad system. He showed the optimal modal split under marginal cost of railroad is set constant. Furthermore, he made clear the optimal congestion charge for automobile commuters. Danielis and Marcucci (2002) expanded Tabuchi's model, and analyzed the congestion charging when railroad fare is set by marginal or average cost. Arnott *et al.* (2000) applied concepts of the classical urban transport economics, and analyzed the second-best congestion charge, amount of railroad facilities and railroad fare. Because these models ignore commuting time distribution, we cannot analyze the change in these distributions when we introduce congestion charging to railroad commuters.

3. FORMULATION OF THE MODEL

3.1 Problem Settings of Our Study

We assume a single road and a railroad connecting a residential area with a central business district (CBD), as shown in **Figure 1**. The road has one bottleneck just before the CBD, where point queue occurs when traffic is over bottleneck's capacity, k(vehicle/min). Furthermore, we

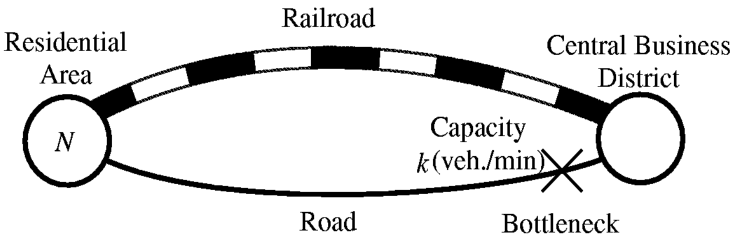

Figure 1. Assumed Commuter Road and Railroad Network

assume that automobile commuters cannot overtake earlier commuters.

All of N commuters drive their vehicle or ride commuter train to CBD where they have their jobs. They can choose both transportation system for commuting, and no other transportation mode can be used. It is assumed that vehicles are driven at constant speed from home to just before the bottleneck, and the duration for this section is equal to constant, w(min), regardless of housing location of the commuters. On the other hand, duration of commuter trains from the residential station to the CBD station is assumed to be constant, κ(min) regardless the railroad congestion level. Furthermore, we ignore access duration from home to the residential station and egress duration from the CBD station to the job site. In addition, we assume that the railroad company must operate commuter trains, whenever commuters hope that they get on these trains. For these reasons, commuters can get on commuter trains without waiting time at the stations. If we carry out congestion charging, we charge the optimal charge at each instant for each transport mode. It is assumed that we charge automobile commuters at the exit of the bottleneck and railroad commuters at the CBD station, respectively.

All firms in the city are located at the CBD and they start working at T_f. Because they do not permit that commuters delay at work start time T_f, all commuters must arrive their firms no later than T_f.

3.2 Disutility of Automobile Commuting

Disutility level, $U_a(q)$, of a representative automobile commuter, q, is defined as the following functions;

$$U_a(q) \quad = \quad e\{m_a(q) - (a(q) + w)\} + c\left\{T_f - a(q)\right\} + \upsilon\{m_a(q) - a(q)\} + \rho_a(q) \qquad (1)$$

where, $a(q)$: home departure time, $m_a(q)$: office arrival time, e: time value of disutility of queuing time (yen/min), c: time value of schedule cost (yen/min), υ: fuel cost per unit of driving time (yen/min), $\rho_a(q)$: congestion charge (yen). In this equation the first term represents the disutility of queuing time at bottleneck, the second term represents the schedule cost for early home departure, the third term represents the fuel cost, and the last term represents the congestion charge if we carry out the congestion charging. In addition, we assume that the fuel cost is in proportion with the duration of driving time, and unit of fuel cost per driving minute, υ, is given by unit fuel price (yen/ℓ) / fuel expense (km/ℓ) $\times$ legal speed (km/min).

3.3 Disutility of Railroad Commuting

Disutility level, $U_r(t)$, of representative railroad commuter is given by the following function of office arrival time, t, which is equal to an arrival time at CBD station by the assumption;

$$U_r(t) \quad = \quad \kappa\,(s(t))^{\eta} + c\left\{T_f - (t - \kappa)\right\} + BC + \rho_r(t) \qquad (2)$$

where, $s(t)$: congestion level of train arriving CBD at t. This is positive and $s(t) = 1$ means that number of commuters on the train equals to the number of seats. Furthermore, we ignore limits of train's capacities. κ: duration of commuter trains from the residential station to the CBD station (min), η: elasticity of the congestion disutility of train, BC: railroad fare level (yen), which is a constant regardless of t, and $\rho_r(t)$: congestion charge (yen). In this equation, the first term represents the disutility of congestion in train, the second term represents the schedule cost for early home departure, and the last two terms are monetary costs.

We define that $u(t)$ is instant supply rate of seats on the trains by the railroad company, and continuous function. Furthermore, $m_r(t)$ is cumulative number of arriving commuters at CBD. We can derive a following equation for marginal number of arriving commuters at CBD, $\dot{m}_r(t)$;

$$\dot{m}_r(t) = s(t)u(t) \tag{3}$$

Railroad operating cost, TRC, is considered as the following integral of instant operating cost, which is a increase function of supply rates $u(t)$ at time t. That integral is done from the arrival time at CBD of the first commuter, T_r, to the arrival time of the last commuter, T_f.

$$TRC = \int_{T_r}^{T_f} \zeta u(t)^{\iota} \mathrm{d}t \tag{4}$$

where, ζ: parameter, $\iota(> 1)$: elasticity of the instant operating cost functions. Let us consider that fare level is regulated to be the average cost. When total number of railroad commuters is N_r, the average fare level per commuter, BC, is given as follows;

$$BC = TRC/N_r \tag{5}$$

Usually in existing studies, it is considered that fixed cost occupies a major part of railroad transport cost. Because this fixed cost is, however, a constant regardless of number of railroad commuters, and it does not influence on the essential structure of our problem, then we ignore the fixed cost.

4. DISTRIBUTIONS OF COMMUTING TIME AND MODAL SPLIT MODEL WITHOUT CONGESTION CHARGING

In this chapter, we consider no-toll situation; $\rho_a(q) = 0$ and $\rho_r(t) = 0$. Every commuter can get the same disutility level in no-toll situation regardless of home departure time and modal choice: because someone could improve disutility by changing his/ her schedule or modal choice if he/ she would get worse disutility than others. Let us consider that the social cost which is defined as the sum of all commuters' disutility is minimized with constant disutility constraint. We call this problem "System Equilibrium (S.E.)" problem.

4.1 Distributions of Commuting Time of Automobile Commuters

Whenever queue occurs at the bottleneck, traffic outflow rate equals k (veh./min). When traffic outflow rate is smaller than k, the schedule cost increases along time. Then, office arrival time $m_a(q)$ satisfies the following equations;

$$m_a'(q) \equiv \frac{\mathrm{d}m_a(q)}{\mathrm{d}q} = \frac{1}{k} \tag{6a}$$

$$m(q) = \frac{q}{k} + \left(T_f - N_a/k\right) \tag{6b}$$

where, N_a: number of automobile commuters.

When number of automobile commuters, N_a, is given, considering equilibrium constraint for the automobile commuter's disutility, $\mathrm{d}U_a(q)/\mathrm{d}q = 0$, we can derive the distribution of home departure time, $a(q)$, as follows;

$$a(q; N_a) = \left(\frac{e+\upsilon}{e+c+\upsilon}\right)\frac{q}{k} + \left(T_f - \frac{N_a}{k} - w\right) \tag{7}$$

Then, we can calculate the equilibrium disutility level of automobile commuter, $U_a(q)$, and the social cost for automobile commuting, SC_a, as follows;

$$U_a(q; N_a) = \frac{c}{k}N_a + (c+\upsilon)\,w \tag{8}$$

$$SC_a(N_a) \equiv U_a(q)\cdot N_a = \frac{c}{k}N_a^2 + (c+\upsilon)\,wN_a \tag{9}$$

4.2 Distributions of Commuting Time of Railroad Commuters

When number of railroad commuters, N_r, is given, considering equilibrium disutility constraint in railroad commuter, $\dot{U}_r(t) \equiv \mathrm{d}U_r(t)/\mathrm{d}t = 0$, we can derive the congestion level of train arriving CBD at t, $s(t)$, as follows;

$$s(t; N_r) = \left(c\left(t - T_r(N_r)\right)\right)^{\frac{1}{\eta}} \tag{10}$$

where, T_r is arrival time at CBD of the first commuter, and function for number of railroad commuters, N_r.

Then, we can calculate the equilibrium disutility of railroad commuter, $U_r(q)$, as follows;

$$U_r(t; N_r) = c\left(T_f - (T_r(N_r) - \kappa)\right) + BC \tag{11}$$

Considering that average fare level, BC, is derived by railroad operating cost, TRC, the social cost for railroad commuting, SC_r, is derived as follows;

$$SC_r(N_r) = c\left(T_f - (T_r(N_r) - \kappa)\right)N_r + \int_{T_r}^{T_f} \zeta u(t)^{\iota}\mathrm{d}t \tag{12}$$

As mentioned above, the social cost minimization for railroad commuting with feasible constraint (3) can be formulated as the following optimal control program;

$$\min_{u(t)} SC_r = c\left(T_f - (T_r(N_r) - \kappa)\right)N_r + \int_{T_r}^{T_f} \zeta u(t)^{\iota}\mathrm{d}t \tag{13a}$$

$$\text{subject to} \quad \dot{m}(t) = s(t)u(t) \tag{13b}$$

$$m(T_r) = 0 \tag{13c}$$

$$m(T_f) = N_r \tag{13d}$$

We can theoretically solve the above program (13) using the optimal control theory, and derive the cumulative number of arriving commuters at CBD, $m_r(t)$, the instant supply rate of railroad company, $u(t)$, and the arrival time at CBD of the first commuter, T_r, as follows;

$$m_r(t; N_r) = \left(\frac{t - T_r(N_r)}{T_f - T_r(N_r)}\right)^{\frac{1}{\eta\theta\psi}} N_r \tag{14}$$

$$u(t; N_r) = \left(\frac{cN_r}{\eta\theta\psi\kappa}\right)\left(\frac{c}{\kappa}(T_f - T_r(N_r)\right)^{-\frac{1}{\eta\theta\psi}} \left(\frac{c}{\kappa}\left(t - T_r(N_r)\right)\right)^{\frac{1}{\eta\theta}} \tag{15}$$

$$T_r(N_r) = T_f - \frac{\kappa}{c}\left(\frac{cN_r}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} \left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \tag{16}$$

where, $\theta \equiv \iota - 1$, $\psi \equiv 1/(1 + \theta + \eta\theta)$, and $\phi \equiv (1 + \eta)(1 + \theta)/(\eta\theta)$.

Then, we can calculate the average fare level, BC, the railroad operating cost, TRC, the equilibrium disutility of railroad commuter, $U_r(t)$, and the social cost for railroad commuting, SC_r, as follows;

$$BC(N_r) = \eta\psi\kappa\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \left(\frac{cN_r}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} \tag{17}$$

$$TRC(N_r) = \eta\psi\kappa N_r\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \left(\frac{cN_r}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} \tag{18}$$

$$U_r(t; N_r) = c\kappa + \kappa\left(1 + \eta\psi\right)\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \left(\frac{cN_r}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} \tag{19}$$

$$SC_r(N_r) \equiv U_r(t) \cdot N_r = c\kappa N_r + \kappa N_r\left(1 + \eta\psi\right)\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \left(\frac{cN_r}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} \tag{20}$$

4.3 Modal Split

Modal split between road and railroad commuting is given by the unique solution N_r^* satisfying the following equation, which means that the disutility levels of railroad commuting (19) is equal to that of automobile commuting (8).

$$(1 + \eta\psi)\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}} \left(\frac{cN_r^*}{\eta\theta\psi}\right)^{\frac{1}{\phi}} + \frac{c}{k}N_r^* + \left\{c\kappa - \frac{c}{k}N - (c + \upsilon)\, w\right\} = 0 \tag{21}$$

where, N_r^* satisfied $N_r = N_r^*$, $N_a = N - N_r^*$.

5. DISTRIBUTIONS OF COMMUTING TIME AND MODAL SPLIT MODEL WITH CONGESTION CHARGING

Let us carry the congestion charging out; $\rho_a(q) \geq 0$, $\rho_r(t) \geq 0$. We can treat disutility of each mode separately, when number of automobile commuters, N_a, and that of railroad commuters, N_r are given. Let us consider the optimal charging which minimizing the social cost on each mode.

5.1 Distribution of Commuting Time of Automobile Commuters and Revenue of Congestion Charge

When number of automobile commuters, N_a, is given, because traffic outflow rate equals k (veh./min) at the bottleneck, nobody can improve his/ her schedule cost to change his/ her office arrival time compared with the S.E. situation. Therefore, we can achieve the optimal situation which social cost becomes minimal by the perfect internalization of externality of congestion via congestion charging. This means the optimal charge equals disutility of congestion in S.E. situation. Then, home departure time, $a(q)$, office arrival time $m_a(q)$, and congestion charge, $\rho_a(q)$ satisfy the following equations;

$$
\begin{aligned}
a(q; N_a) + w &= m_a(q; N_a) = \frac{q}{k} + \left(T_f - \frac{N_a}{k}\right) &\quad (22)\\
\rho_a(q) &= \frac{c}{k} q &\quad (23)
\end{aligned}
$$

Then, we can calculate the social cost for automobile commuting, SC_a^o, and the revenue of congestion charge, PR_a, as follows;

$$
\begin{aligned}
SC_a^o(N_a) &= \frac{c}{2k} N_a^2 + (c + \upsilon)\, w N_a &\quad (24)\\
PR_a(N_a) &= \frac{1}{2} \frac{c}{k} N_a^2 &\quad (25)
\end{aligned}
$$

When the revenue is not returned to the commuters, disutility of the automobile commuters, U_a^o, is shown as follows;

$$
U_a^o(N_a) = \frac{c}{k} N_a + (c + \upsilon)\, w \quad (26)
$$

This equation shows disutility of automobile commuting is equal to that of S.E. situation if number of automobile commuters is same.

5.2 Distribution of Commuting Time of Railroad Commuters and Revenue of Congestion Charge

From social point of view, the congestion charge is revenue, and payment of commuters is compensated. Therefore, social cost for railroad commuting with the congestion charging, SC_r^o, is given as follows;

$$
\begin{aligned}
SC_r^o &= \int_{T_r}^{T_f} \left[\dot{m}_r(t) \left(U_r(t) - \rho_r(t)\right)\right] \mathrm{d}t \\
&= \int_{T_r}^{T_f} \left[s(t)u(t) \left\{\kappa s(t)^\eta + c\left(T_f - (t - \kappa)\right)\right\} + \zeta u(t)^\iota\right] \mathrm{d}t &\quad (27)
\end{aligned}
$$

As mentioned above, the social cost minimizing program for railroad commuting with the congestion charging can be formulated as the following optimal control program with control variables, $s(t)$, and $u(t)$;

$$
\begin{aligned}
\min_{s(t),u(t)} SC_r^o &= \int_{T_r}^{T_f} \left[s(t)u(t) \left\{\kappa s(t)^\eta + c\left(T_f - (t - \kappa)\right)\right\} + \zeta u(t)^\iota\right] \mathrm{d}t &\quad (28a)\\
s.t. \quad & \dot{m}_r(t) = s(t)u(t) &\quad (28b)\\
& m_r(T_r) = 0 &\quad (28c)\\
& m_r(T_f) = N_r &\quad (28d)
\end{aligned}
$$

We can theoretically solve the above program (28) using the optimal control theory, and derive the cumulative number of arriving commuters at CBD, $m_r(t)$, the congestion level of train, $s(t)$, the instant supply rate of railroad company, $u(t)$, and the arrival time at CBD of the first commuter, T_r, as follows;

$$\begin{aligned}
m_r(t;N_r) &= \frac{\zeta\theta}{c}\left(\frac{\eta\kappa}{\zeta\iota}\right)^{\frac{1+\theta}{\theta}}\left(\frac{c\,(t-T_r(N_r))}{\kappa\,(1+\eta)}\right)^{\phi} && (29)\\
s(t;N_r) &= \left(\frac{c\,(t-T_r(N_r))}{\kappa\,(1+\eta)}\right)^{\frac{1}{\eta}} && (30)\\
u(t;N_r) &= \left(\frac{\eta\kappa}{\zeta\iota}\right)^{\frac{1}{\theta}}\left(\frac{c\,(t-T_r(N_r))}{\kappa\,(1+\eta)}\right)^{\frac{1+\eta}{\eta\theta}} && (31)\\
T_r(N_r) &= T_f-\frac{\kappa\,(1+\eta)}{c}\left(\frac{cN_r}{\zeta\theta}\right)^{\frac{1}{\phi}}\left(\frac{\zeta\iota}{\eta\kappa}\right)^{\frac{\eta}{1+\eta}} && (32)
\end{aligned}$$

Then, we can calculate the congestion charge, $\rho_r(t)$, the average fare level, BC, the railroad operating cost, TRC, the social cost for railroad commuting, SC_r^o, and the revenue of congestion charge, PR_r, as follows;

$$\begin{aligned}
\rho_r(t;N_r) &= \frac{c\eta}{1+\eta}\,(t-T_r(N_r))\quad (=\eta s(t)^{\eta}) && (33)\\
BC(N_r) &= \zeta^{\frac{1}{\phi\theta}}\frac{\phi}{1+\phi}\left(\frac{\eta}{\iota}\right)^{\frac{1}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}} && (34)\\
TRC(N_r) &= \zeta^{\frac{1}{\phi\theta}}N_r\frac{\phi}{1+\phi}\left(\frac{\eta}{\iota}\right)^{\frac{1}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}} && (35)\\
SC_r^o(N_r) &= c\kappa N_r+\zeta^{\frac{1}{\phi\theta}}N_r\,(1+\eta)\,\frac{\phi}{1+\phi}\left(\frac{\iota}{\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}} && (36)\\
PR_r(N_r) &= \zeta^{\frac{1}{\phi\theta}}N_r\frac{\phi\kappa\eta}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}} && (37)
\end{aligned}$$

When the revenue is not returned to the commuters, disutility of the railroad commuters, U_r^o, is shown as follows;

$$U_r^o(N_r) = c\kappa+\zeta^{\frac{1}{\phi\theta}}\,(1+2\eta)\,\frac{\kappa\phi}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}} \qquad (38)$$

This equation shows that disutility of railroad commuting is lower than that of S.E. situation if number of railroad commuters is same, because period of commuting time becomes longer in order to decrease peak congestion level under the congestion charging. Therefore, when the revenue is not returned to the commuters, disutility level becomes worse by introduction of the congestion charging.

5.3 Congestion Charge and Return Schemes and Modal Split

Let us consider that commuters are carried congestion charge and return schemes out on both modes, using above models. In addition, we assume that full revenue is returned to commuters, and the return each commuter is equal to others in same mode.

(I) Charge for Automobile Commuters

In case of charge for automobile commuters (**case** *i*), the distributions of automobile commuting time represent Eq. (22). The distribution of railroad commuting time represents Eq. (14) in S.E. situation. When rate x of the revenue (25) distribute to railroad commuters, and rate $(1-x)$ of that distribute to automobile commuters, utilities of each mode after the return represent as follows;

$$\begin{aligned} U_a^i(x; N_a) &= U_a^o(N_a) - (1-x)\, PR_a/N_a && (39a) \\ U_r^i(x; N_r) &= U_r^e(N_r) - x \cdot PR_a/N_r && (39b) \end{aligned}$$

where, $N_a + N_r = N$. From first-order condition for N_r, followings equations are derived;

$$\begin{aligned} \frac{\partial U_a^i(x; N_r)}{\partial N_r} &= -\frac{c}{2k}(1+x) < 0 && (40a) \\ \frac{\partial U_r^i(x; N_r)}{\partial N_r} &= \left(\frac{1+\eta\psi}{\phi}\right)\left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}}\left(\frac{c}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} N_r^{\frac{1}{\phi}-1} + x\frac{c}{2k}\left(\left(\frac{N}{N_r}\right)^2 - 1\right) > 0 && (40b) \end{aligned}$$

Then, equilibrium modal split is derived from $N_r = N_r^*$, $N_a = N - N_r^*$ satisfied $U_a^i(x; N_a^*) = U_r^i(x; N_r^*)$, and we can derive unique solution as following equation;

$$(1+\eta\psi)\, N_r^* \left(\frac{\zeta}{\eta\psi\kappa}\right)^{\frac{1}{\phi\theta}}\left(\frac{cN_r^*}{\eta\theta\psi\kappa}\right)^{\frac{1}{\phi}} - \frac{1}{2}\frac{c}{k}(N - N_r^*)(xN + N_r^*) + \{c\kappa - (c+\upsilon)\, w\}\, N_r^* = 0 \quad (41)$$

(II) Charge for Railroad Commuters

In case of charge for railroad commuters (**case** *ii*), the distribution of railroad commuting time represents Eq. (29). The distributions of automobile commuting time represent Eq. (6b), (7) in S.E. situation. When rate x of the revenue (37) distribute to railroad commuters, and rate $(1-x)$ of that distribute to automobile commuters, utilities of each mode after the return represent as follows;

$$\begin{aligned} U_a^{ii}(x; N_a) &= U_a^e(N_a) - (1-x)\, PR_r/N_a && (42a) \\ U_r^{ii}(x; N_r) &= U_r^o(N_r) - x \cdot PR_r/N_r && (42b) \end{aligned}$$

From first-order condition for N_r, followings equations are derived;

$$\begin{aligned} \frac{\partial U_a^{ii}(x; N_r)}{\partial N_r} &= -\frac{c}{k} - (1-x)\frac{\zeta^{\frac{1}{\phi\theta}}\kappa\eta}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}}\left(\frac{(1+\phi)\,N - N_r}{(N-N_r)^2}\right) < 0 && (43a) \\ \frac{\partial U_r^{ii}(x; N_r)}{\partial N_r} &= \zeta^{\frac{1}{\phi\theta}}\frac{\kappa}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{c}{\theta}\right)^{\frac{1}{\phi}} N^{\frac{1}{\phi}-1}(1+\eta\,(2-x)) > 0 && (43b) \end{aligned}$$

Then, equilibrium modal split is derived from $N_r = N_r^*$, $N_a = N - N_r^*$ satisfied $U_a^{ii}(x; N_a^*) = U_r^{ii}(x; N_r^*)$, and we can derive unique solution as following equation;

$$\begin{aligned} &\zeta^{\frac{1}{\phi\theta}}\frac{\phi\kappa}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r^*}{\theta}\right)^{\frac{1}{\phi}}\{(1+\eta\,(2-x))\,N - (1+\eta)\,N_r^*\} \\ &\qquad -\frac{c}{k}(N - N_r^*)^2 + \{c\kappa - (c+\upsilon)\,w\}\,(N - N_r^*) = 0 \qquad (44) \end{aligned}$$

(III) Charge for Both Automobile and Railroad Commuters

In case of charge both automobile and railroad commuters (**case** *iii*), the distributions of automobile commuting time represent Eq. (22), and that of railroad commuting time represents Eq. (29). When rate x of the revenue (25)+(37) distribute to railroad commuters, and rate $(1-x)$ of that distribute to automobile commuters, utilities of each mode after the return represent as follows;

$$U_a^{iii}(x; N_a) = U_a^o(N_a) - (1-x)(PR_a + PR_r)/N_a \tag{45a}$$
$$U_r^{iii}(x; N_r) = U_r^o(N_r) - x(PR_a + PR_r)/N_r \tag{45b}$$

From first-order condition for N_r, followings equations are derived;

$$\frac{\partial U_a^{iii}(x; N_r)}{\partial N_r} = -(1+x)\frac{c}{2k} - (1-x)\frac{\zeta^{\frac{1}{\phi\theta}}\kappa\eta}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r}{\theta}\right)^{\frac{1}{\phi}}\left(\frac{(1+\phi)N - N_r}{(N-N_r)^2}\right) < 0 \tag{46a}$$

$$\frac{\partial U_r^{iii}(x; N_r)}{\partial N_r} = x\frac{c}{2K}\left(\left(\frac{N}{N_r}\right)^2 - 1\right) + \zeta^{\frac{1}{\phi\theta}}\frac{\kappa}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{c}{\theta}\right)^{\frac{1}{\phi}} N^{\frac{1}{\phi}-1}(1+\eta(2-x)) > 0 \tag{46b}$$

Then, equilibrium modal split is derived from $N_r = N_r^*$, $N_a = N - N_r^*$ satisfied $U_a^{iii}(x; N_a^*) = U_r^{iii}(x; N_r^*)$, and we can derive unique solution as following equation;

$$\zeta^{\frac{1}{\phi\theta}}\frac{\kappa\phi}{1+\phi}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{c}{\theta}\right)^{\frac{1}{\phi}}(N_r^*)^{1+\frac{1}{\phi}}\{(1+\eta(2-x))N - (1+\eta)N_r^*\} - \frac{1}{2}\frac{c}{k}(N-N_r^*)^2(xN + N_r^*) + \{c\kappa - (c+\upsilon)w\}N_r^*(N-N_r^*) = 0 \tag{47}$$

(IV) First-Best Charge

In this section, let us consider that the first-best problem which is minimized the social cost ((24)+(26)). We call this problem system optimum (S.O.) one. The optimal modal split is derived as following equation;

$$\kappa(1+\eta)\zeta^{\frac{1}{\phi\theta}}\left(\frac{\iota}{\kappa\eta}\right)^{\frac{\eta}{1+\eta}}\left(\frac{cN_r^o}{\theta}\right)^{\frac{1}{\phi}} + \frac{c}{k}N_r^o + \left\{c\kappa - \frac{c}{k}N - (c+\upsilon)w\right\} = 0$$

6. NUMERICAL EXAMPLE

In this section, we show numerical examples to use the models in the last section. Parameter values for the numerical examples below are given as follows; $c = 10$(yen/min), $e = 20$(yen/min), $k = 110$(veh./min), $\upsilon = 6.67$(yen/min), $w = 30$(min), $\eta = 4.5$, $\iota = 3.1$, $\zeta = 0.0008$, $\kappa = 20$(min), $N = 50{,}000$, $T_f = 9:00$.

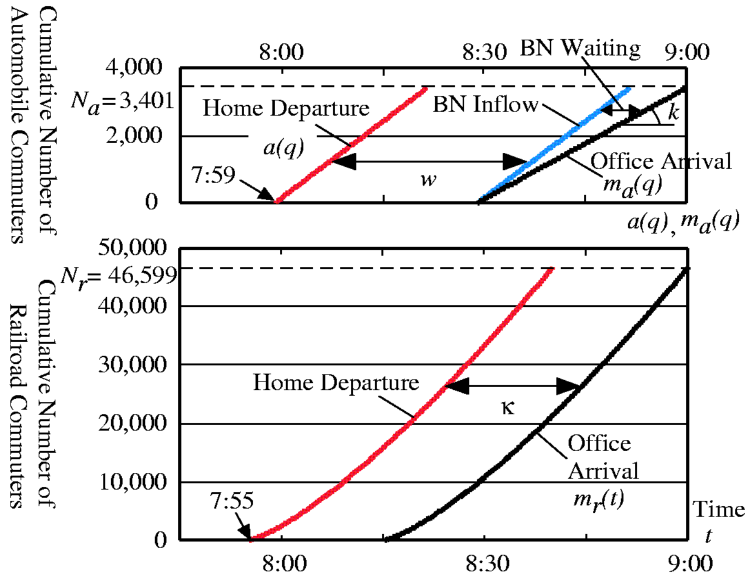

Figure 2. Distributions of Commuting Time in S.E.

6.1 Distributions of Commuting Time

Figure 2 shows the schedule pattern and the modal split without the congestion charging (S.E.). In this numerical simulation, $N_a^* = 3{,}401$ and $N_r^* = 46{,}599$. Home departure time of the first automobile commuter is 7:59, and that of the first railroad commuter is 7:55.

On the other hand, **Figure 3** shows the schedule pattern and the modal split with the optimal congestion charging (S.O.). In addition, they show distribution of congestion charge. In this numerical simulation, $N_a^o = 5{,}158$ and $N_r^o = 44{,}842$, then, automobile commuters increase 1,757. Furthermore, home departure time of the first automobile commuter is 7:43, and that of the first railroad commuter is 7:23. Because congestion levels of trains are decreased, the period of railroad commuting time is longer than S.E. situation in spite of number of railroad commuters is decrease.

6.2 Economic Evaluation for Congestion Charge and Return Schemes

First, we analyze the return only to automobile commuters (**case (*I*)**) which is $x = 0$, and only to railroad commuters (**case (*II*)**) which is $x = 1$, as return schemes. Therefore, we consider the combination schemes of charges (**case (*i*), (*ii*)**) and returns (**case (*I*), (*II*)**).

Figure 4 shows disutility levels each mode when modal split is given under each charge and return scheme. Furthermore, equilibrium modal split (solution) is shown as a white circle under each charge and return scheme. In addition, black circle shows S.E. and S.O. situation in order to compare with each scheme.

In S.E. situation, equilibrium disutility is −809 (yen).

When congestion charge and return schemes are carried out only to automobile commuters $\{iI\}$, number of automobile commuters is increase. Because marginal disutility of railroad commuter is not elastic, equilibrium disutility level does not be improved as compared with that of S.E. On the other hand, when congestion charge and return schemes are carried out only to railroad commuters $\{iiII\}$, number of railroad commuters is increase. Because marginal disutility of automobile commuter is, then, elastic, equilibrium disutility level is improved effective as compared with that of S.E.

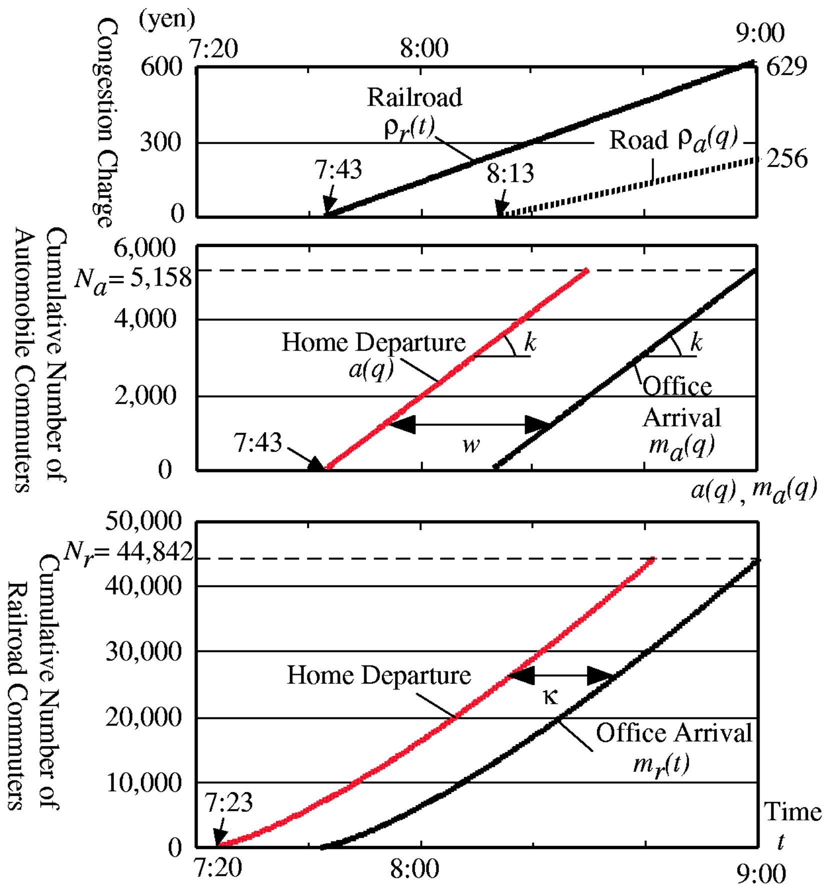

Figure 3. Distributions of Commuting Time and Congestion Charge in S.O.

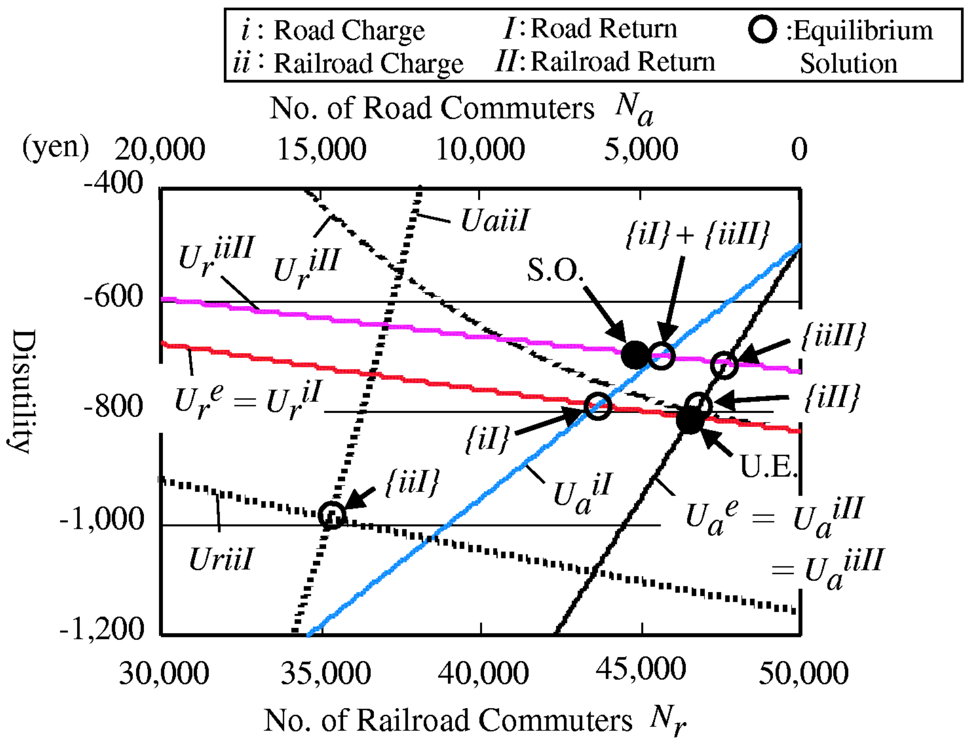

Figure 4. Disutility Levels under Each Charge and Return Schemes

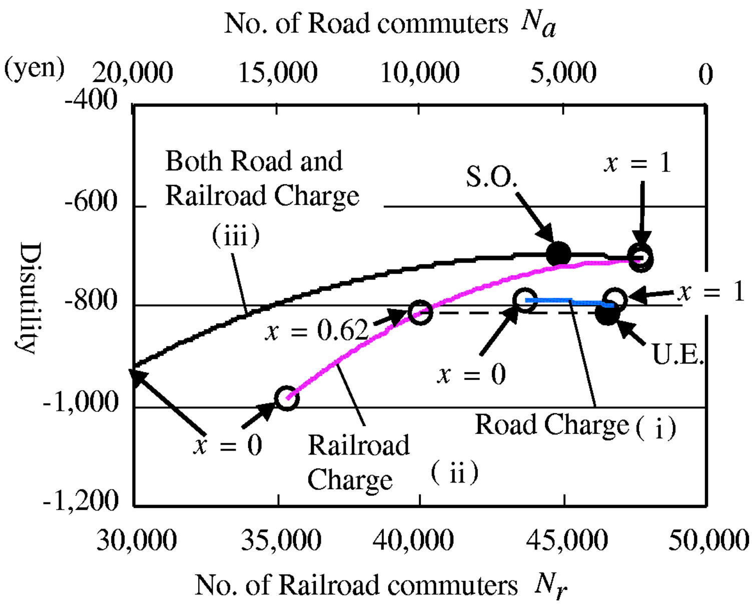

Figure 5. Equilibrium Disutility Levels Each Return Rate

When congestion charge is carried out to automobile commuters and return is carried out to railroad commuters {*iII*}, modal split and equilibrium disutility level do not change, because the amount of the revenue is small and the return of each railroad commuter is very low. On the other hand, when congestion charge is carried out to railroad commuters and return is carried out to automobile commuters {*iiI*}, number of automobile commuters is increase drastically, and equilibrium disutility level is worse.

Finally, we consider that the schemes of revenue are divided to automobile and railroad commuters ($0 \leq x \leq 1$). **Figure 5** shows equilibrium disutility and modal split each x.

When congestion charge is carried out to automobile commuters {*i*}, modal split is change corresponding to the return rate x, but disutility level does not change. On the other hand, when congestion charge is carried out to railroad commuters {*ii*}, equilibrium disutility level is improved if x is over than 0.62 as compared with S.E. situation.

7. CONCLUSION

We have proposed and solved the optimal control models to simultaneously determine the distribution of home departure time and the modal choice between road and railroad with/without the congestion charging. We theoretically have made clear the optimal congestion charge each time of day for each transport mode. Furthermore, we have analyzed influence of return scheme on the modal choice.

From results of numerical simulation using above models, we have shown that scheme of return to other mode does not improve utility. Another implication is that the charging schemes to railroad commuters is more effective to improve commuter's disutility and to reduce the number of automobile commuters in contrast with the road charging scheme which is examined by many local governments.

It should be noted, however, that we set very strong assumptions such as commuter's behavior neglects personal heterogeneity, road and railroad line are both only single line, and access and egress time are ignored. In future works, we should relax these unrealistic assumptions. However, from the viewpoint of travel behavior modeling, these models seem far from the reality. But owing to the simplifications, we get analytical tractability. If we combine more reliable parameter values based on behavior analysis, we can expect the market equilibrium and effect of policies can be more precisely calculated. Especially, we need to correctly estimate the time value in order to success the congestion charging.

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